**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Solution- Since we have mean=45 and Standard deviation = 8**

**To finish the work in one hour (x<=50), so the question to find is (x>50)**

**Which is (x>50) =1-(x<=50).**

**Therefore X=50**

**By using the formula, P(X-mean/Standard Deviation)**

**P (50-45/8) = P (0.625)**

**By using Z table**

**= 0.732**

**= 73.2%**

**Therefore, the probability the service manager will not meet his demand will be**

**=100-73.2=26.6% or 0.266**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Solution –A) Mean= 38, Standard Deviation= 6**

**Older than, P(X>44) = 1-P(X<=44)**

**Z= (44-38)/6= P (1) = 0.84134 i.e. 84.134 % (Using Z table)**

**Therefore, P(X>44) = 100-84.134=15.86%**

**Z= (38-38)/6=P (0) =50%**

**So the Probability of number of employees between 38-44 years of age**

**=83.134%-50%=34.13%**

**Hence the statement is FALSE**

**B) P(X<30) =Z (30-38)/6= -1.33= 9.15 %( Using Z table) approx. to 36 out of 400**

**Hence the statement is TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Solution- As we know that,**

**if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .**

**Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).**

**Therefore in the question**

**2X1~ N(2 u,4 σ^2) and**

**X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )**

**2X1-(X1+X2) = N( 4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Solution- In this we have to first work in the reverse order I.e.**

**Probability of getting the value between a and b is 0.99**

**Then, Probability of getting the values outside of a and b is =1-0.99=0.01**

**For the outside**

**P (Left of a)=0.01/2=-0.005**

**P (Right of b)=0.01/2=+0.005**

**Since we want the value of X, We can take it out by finding Z**

**Probability of 0.005 has Z value as -2.57 (From Z table/ Google)**

**Z (-0.005)\*20+100 = - (-2.57)\*20+100 = X (151.4)**

**Z (+0.005)\*20+100 = (-2.57)\*20+100 = X (48.6)**

**Ans is D**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Solution- Mean Profit**

**= Mean 1 + Mean 2=5+7=12**

**Therefore, In Rs the mean profit is= 45\*12=540 Million ($= million)**

**Standard Deviation Profit= Sqrt(9+16)=5**

**Therefore, in Rs the SD profit is = 45\*5=225 Million ($= million)**

1. **By using Formula in Python**

**stats.norm.interval(0.95,540,225)**

**Range is Rupee (99.00810347848784, 980.9918965215122) in Millions**

1. **To compute 5th Percentile, we use the formula X=μ + Zσ**

**For 5% Z value is= -1.645**

**X= 540 + (-1.645)\*(225)**

**Using Formula np.round (X,) (Rounds the number to nearest integer value.**

1. **Using Formula for Normal Distribution in Python**

**Division 1) stats.norm.cdf(0,5,3)**

**0.0477903522728147**

**Division 2) stats.norm.cdf(0,7,4)**

**0.040059156863817086**

**First Division of the company has higher probability of making loss**